Experiment No. 9

Z-Transform and Inverse Z-Transform Analysis

Objectives

1. To learn about the implementation of Z-Transform using MATLAB.
2. To determine and plot the poles and zeroes on the z-plane.
3. To be able to understand and implement Inverse Z-Transform.

Equipment

1. PC
2. Installed MATLAB

Introduction

Laplace transform methods are widely used for analysis in linear systems. Laplace transform methods are used when a system is described by a linear differential equation, with constant coefficients. However: There are numerous systems that are described by difference equations - not differential equations - and those systems are common and different from those described by differential equations.

1. Systems that satisfy difference equations include things like:

Computer controlled systems - systems that take measurements with digital I/O boards or GPIB instruments, calculate an output voltage and output that voltage digitally. Frequently these systems run a program loop that executes in a fixed interval of time.

Other systems that satisfy difference equations are those systems with Digital Filters - which are found anywhere digital signal processing - digital filtering is done. That includes:

2. Digital signal transmission systems like the telephone system.

Systems that process audio signals. For example, a CD contains digital signal information, and when it is read off the CD, it is initially a digital signal that can be processed with a digital filter.

At this point, there are an incredible number of systems we use every day that have digital components which satisfy difference equations.

In continuous systems Laplace transforms play a unique role. They allow system and circuit designers to analyze systems and predict performance, and to think in different terms - like frequency responses - to help understand linear continuous systems. They are a very powerful tool that shapes how engineers think about those systems. Z-transforms play the role in sampled systems that Laplace transforms play in continuous systems.

1. In continuous systems, inputs and outputs are related by differential equations and Laplace transform techniques are used to solve those differential equations.

2. In sampled systems, inputs and outputs are related by difference equations and Z-transform techniques are used to solve those differential equations.

In continuous systems, Laplace transforms are used to represent systems with transfer functions, while in sampled systems, Z-transforms are used to represent systems with transfer functions.

Z-Transform

A discrete-time signal is defined in the discrete time domain n; that is, it is given by a function:

$$f[n]\,, n \in \mathbb{Z}$$

Z-Transform is denoted by the symbol $Z\{.\}$ and expresses a signal in z-domain, i.e., the signal is given by a function $F(z)$. The mathematical expression is

$$F(z) = Z\{f[n]\}$$

In other words, the z-transform of a function $f(n)$ is $F(z)$. The mathematical expression of the two- sided (or-bilateral) z-transform is:

$$F(z) = Z\{f[n]\} = \sum_{n=-\infty}^{\infty} f[n]z^{-n}$$

Where z is a complex variable. Setting the lower limit of the sum from minus infinity to zero yields the one-sided (or unilateral) z-transform whose mathematical expression is:

$$F(z) = Z\{f[n]\} = \sum_{n=0}^{\infty} f[n[z^{-n}$$

Inverse Z-Transform

In order to return from the z-domain back to the discrete-time domain, the inverse z-transform is applied. The inverse z-transform is denoted by the symbol $Z^{-1}\{.\}$; that is, one can write

$$f[n] = Z^{-1}\{F(z)\}$$

The mathematical expression of the inverse z-transform is:

$$x(n) = Z^{-1}\{F(z)\} = \frac{1}{2\pi j}\oint X(z)\, z^{n-1} dz$$

Command $ztrans$ **and** $iztrans$

In MATLAB, the z-transform $F(z)$ of a sequence $f[n]$ is computed easily by using the command "*ztrans"*. Moreover, the inverse z-transform of a function $F(z)$ is computed by using the command *"iztrans"*. Before using these two commands, the declaration of the complex variable z and of the discrete time n as symbolic variables is necessary. Recall that in order to define a symbolic variable, the command syms is used. The commands "*ztrans"* and "i*ztrans"* are used exactly in same way as the commands "*laplace"*. and "*ilaplace"* are used to compute the Laplace and inverse Laplace transforms of a function. Finally, we note that the command

"*ztrans"* computes the unilateral z-transform.

The function $f(n) = 2^n u[n]$ and $F(z) = \frac{z}{z-2}$ are a z-transform pair. In other words, the z-transform of earlier one is later one and inverse z-transform of later one is earlier one. A z-transform pair is denoted by $x[n] \leftrightarrow X(z)$. At the moment, we discuss alternative syntaxes of the commands "*ztrans"* and "*ztrans"*. The most effective, and in the same time simple, syntax of the command "*ztrans"* is "*ztrans(f , z)"*. In this way, the z-transform of a sequence $f[n]$ is expressed with the second input argument (here is z) as the independent variable. Using this syntax is optimal and makes possible the computation of the z-transform in every case. Such a case is when z-transform of a constant function has to be computed.

Region of Convergence

The z-transform of a sequence does not always exist. The region of convergence (ROC) of the z- transform of a sequence $x[n]$ is the range of z for which the z-transform of $x[n]$ is not infinite. For example, the z-transform $X(z)$ of the sequence $x[n] = 0.8^n u[n]$ is computed as:

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} 0.8^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{0.8}{z}\right)^n$$

Thus $X(z)$ converges; that is,

$$X(x) < \infty \qquad \text{if} \qquad \left|\frac{0.8}{z}\right| < 1 \implies |z| > 0.8$$

Therefore, the ROC of the z- transform $X(x)$ of the signal $x[n] = 0.8^n u[n]$ is $|z| > 0.8$.

Activity-1: Use MATLAB and compute the z-transform of the basic functions given below.

a. $x(t) = u[n]$

```
y=sym(1);
u=heaviside(y);
X=ztrans(u);
disp('X(z)= ');
pretty(X)
```

b. $x(t) = a^n u[n]$

```
syms n a
y=sym(1);
u=heaviside(y);
x=(a^n)*u;
X=ztrans(x);
disp('X(z)= ')
pretty(X)
```

Partial Fraction Expansion of a Rational Function

The z-transform of a sequence is usually expressed as a rational function of z, i.e., it is written as a ratio of two polynomials of z. The mathematical expression is:

$$X(z) = \frac{B(z)}{A(z)} = \frac{b_m z^m + b_{m-1} z^{m-1} + b_{m-2} z^{m-2} + \cdots + b_1 z + b_0}{a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_1 z + a_0}$$

Where a_i, b_i are real numbers. In order to express the function $X(z)$ in a partial fraction form, we follow a similar approach to that followed in the Laplace transform experiment. A signal in rational form can be expressed in partial fraction form by using the command residue. The command residue is used to confirm the result of the previous example.

Activity-2: Determine the poles and zeros of the function given below. Also draw the pole-zero plot.

$$X(z) = \frac{1 - 0.25z^{-1}}{1 + z^{-1} + 2z^{-2}}$$

```
clear
clc
num=[1 -0.25];
denum=[1 1 2];
[z,p,k]=tf2zp(num,denum)
zplane(num,denum);grid
text(real(z)+0.1,imag(z),"Zero");
text(real(p)+0.1,imag(p),"Pole");
```

Activity-3: Use MATLAB and compute the inverse z-transform of the function given below.

$$X(z) = \frac{z}{z - 0.9} + \frac{z}{z - 1}$$

```
clear
clc
syms z
X=(z/(z-0.9))+(z/(z-1));
x=iztrans(X);
disp('x[n]= ');
pretty(x)
```

Activity-4: Use MATLAB and compute the inverse z-transform of the functions given below by computing partial fraction using "*residuez*" command.

$$X(z) = \frac{8 - 4z^{-1}}{z^{-2} + 6z^{-1} + 8}$$

```
clear
clc
close all
syms z
num=[8 -4];
denum=[8 6 1];
[zeros,poles,gain]=tf2zp(num,denum)
zplane(num,denum);grid
text(real(zeros)+0.02,imag(zeros),"Zero");
text(real(poles)+0.02,imag(poles),"Pole");
[r,p,k]=residuez(num,denum)
X=((z*r(1))/(z-p(1)))+((z*r(2))/(z-p(2)))
x=iztrans(X);
disp('x[n]= ');
pretty(x)
```

Lab Tasks (Psychomotor)

1. Compute the z-transform of the following function using MATLAB.
   a. $\boldsymbol{x[n] = u[n] + n\,u[n] + a^n u[n] + n a^n u[n]}$
   b. $\boldsymbol{x[n] = cos(\omega n) u[n] + sin(\omega n)\, u[n]}$
2. Determine and plot the poles and zeros of a user defined 5th order function $X(z)$.
3. Compute the inverse z-transform of the function given below by computing partial fraction using MATLAB. Also determine the zeros & poles and plot them.

$$X(z) = \frac{z^{-1} + \frac{1}{2}z^{-2}}{1 + \frac{3}{5}z^{-1} + \frac{2}{25}z^{-2}}$$

Lab Tasks (Cognitive)

Q. No.	Cognitive Assessment Question	Marks
1	For x[n]=u[n]+nu[n], identify the two basic sequence components and state the corresponding standard Z-transform forms.	2
2	For X(z)=(z−2)(z+1)/[(z−3)(z−4)], identify the zeros and poles from the factored expression.	2
3	Explain how the pole locations of a rational X(z) help in analyzing the behavior of a discrete-time system.	2
4	For a rational Z-transform that requires partial fractions, explain why decomposition into simpler terms facilitates the inverse Z-transform.	2
5	Given a right-sided sequence with a pole at z=0.5, state whether the corresponding ROC lies inside or outside the pole and justify your choice.	2

Lab Exercise and Summary

Summary should cover Introduction, Procedure, Data Analysis and Evaluation.

Student's Signature: _____ Date: _____

LABORATORY SKILLS ASSESSMENT (Cognitive)

Obtained Marks	/10

Instructor's Signature: _____ Date: _____

LABORATORY SKILLS ASSESSMENT (Psychomotor)

Psychomotor Activity 1 — Experimental Setup

S. No	Criteria	Performance	Description	Marks
1	MATLAB Symbolic/Z-Transform Setup (4)	Level 4	Correctly prepares MATLAB with symbolic variables and the required Z-transform environment.	4
		Level 3	Correctly prepares MATLAB with minor errors or assistance.	3
		Level 2	Partially prepares the required environment and requires considerable assistance.	2
		Level 1	Shows limited ability to prepare MATLAB and requires substantial assistance.	1
2	Z-Transform File Setup (4)	Level 4	Correctly organizes the script for forward transform, inverse transform and pole-zero calculations.	4
		Level 3	Organizes the script for forward transform, inverse transform and pole-zero calculations with minor errors or assistance.	3
		Level 2	Partially organizes the script for forward transform, inverse transform and pole-zero calculations with considerable assistance.	2
		Level 1	File setup is largely incomplete and requires substantial assistance.	1
3	Sequence/ Transform Parameters (2)	Level 2	Correctly defines n, z, coefficients, orders and other required parameters.	2
		Level 1	Defines n, z, coefficients, orders and other required parameters with errors or assistance.	1
Total Obtained				/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Psychomotor Activity 2 — Implementation + Results

S. No	Criteria	Performance	Description	Marks
1	Z-Transform Implementation & Safe Working Practice (4)	Level 4	Correctly implements the assigned Z-transform, inverse transform and pole-zero calculations independently and follows proper laboratory practices.	4
		Level 3	Implements the assigned work with minor errors or assistance and generally follows proper laboratory practices.	3
		Level 2	Partially implements the assigned work and requires considerable assistance or repeated guidance.	2
		Level 1	Implementation is largely incomplete, requires substantial assistance, or shows poor adherence to proper laboratory practices.	1
2	Execution & Verification (2)	Level 2	Correctly executes symbolic and plotting commands and checks the results.	2
		Level 1	Executes symbolic and plotting commands and checks the results with minor errors or assistance.	1
3	Transform & Pole-Zero Results (4)	Level 4	Produces correct transform expressions, inverse sequence and pole-zero plot.	4
		Level 3	Produces mostly correct transform expressions, inverse sequence and pole-zero plot with minor errors or omissions.	3
		Level 2	Produces partially correct results/plots with noticeable errors or omissions.	2
		Level 1	Results/plots are largely incorrect or incomplete.	1
	Total Obtained			/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Instructor's Signature: _____ **Date: _____**

LABORATORY SKILLS ASSESSMENT (Affective)

Affective Activity — Lab Report + Viva Voce

S. No	Criteria	Performance	Description	Marks
1	**Lab Report Completeness & Organization (4)**	Level 4	Report is complete, well-organized, properly presented, and demonstrates responsible completion of the assigned Signals and Systems laboratory work.	**4**
		Level 3	Report is complete with minor omissions or organizational issues and demonstrates satisfactory responsibility for the assigned work.	**3**
		Level 2	Report is partially complete with several omissions and demonstrates limited responsibility for the assigned work.	**2**
		Level 1	Report has major omissions or poor organization and demonstrates minimal responsibility for the assigned work.	**1**
2	**Code, Results & Conclusion (2)**	Level 2	MATLAB code, results/plots, and conclusion are correctly presented and discussed, demonstrating ownership of the assigned work.	**2**
		Level 1	MATLAB code, results/plots, or conclusion are presented with minor errors or omissions.	**1**
3	**Viva Voce (4)**	Level 4	Clearly explains the assigned MATLAB code, signal/system concepts, analysis, results, and plots, demonstrating individual responsibility, participation, and effective communication.	**4**
		Level 3	Explains most of the assigned work and results correctly, with minor difficulty, and demonstrates good individual participation.	**3**
		Level 2	Explains basic aspects of the assigned work with some prompting and demonstrates basic participation.	**2**
		Level 1	Demonstrates limited understanding or involvement and requires substantial prompting to explain the assigned work.	**1**
	Total Obtained			/10

The student has not submitted the report and has not appeared in the Viva Voce.

Criteria	Description	Marks
Lab Report and Results Not Submitted	Student has not submitted the report and results file.	**0**
Absent in Viva Voce	Student is absent from the viva voce.	**0**

Instructor's Signature: _____ **Date:** _____