Experiment No. 8

Laplace Transform and Inverse Laplace Transform Analysis

Objectives

1. To learn about the implementation of Laplace Transform and its properties using MATLAB.
2. To be able to understand and implement Inverse Laplace Transform.

Equipment

1. PC
2. Installed MATLAB

Laplace Transform

The purpose of the Laplace Transform is to transform ordinary differential equations (ODEs) into algebraic equations, which makes it easier to solve ODEs. However, the Laplace Transform gives one more than that: it also does provide qualitative information on the solution of the ODEs (the prime example is the famous final value theorem).

A continuous-time signal described in the time domain t is a signal given by a function $f(t)$. As previously mentioned, the Laplace transform expresses a signal in the complex frequency domain s(or s domain); that is, a signal is described by a function $F(s)$. Laplace transform is denoted by the symbol $L\{.\}$; that is, one can write:

$$F(s) = L\{f(t)\}$$

In other words, the Laplace transform of a function $f(t)$ is a function $F(s)$. An alternative way of writing is:

$$f(t) \overset{L}{\rightarrow} F(s)$$

There are two available forms of Laplace transform. The first is the two-sided (or bilateral) Laplace transform where the Laplace transform F(s) of a function $f(t)$ is given by:

$$F(s) = L\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-st}dt$$

Variable s is a complex-valued number, and thus can be written as $s = \sigma + j\omega$. This relationship reveals the association between the Laplace transform and the Fourier transform. More specially, substituting s by $j\omega$ yields the Fourier transform $X(\omega)$of $x(t)$. Setting the lower limit of the integral to zero yields the one-sided (or unilateral) Laplace transform, which is described by the relationship:

$$F(s) = L\{f(t)\} = \int_{0}^{\infty} f(t)e^{-st}dt$$

Command *laplace*

The computation of the integrals can sometimes be quite hard. This is the reason that already computed Laplace transform pairs are used to compute the Laplace transform

and the inverse Laplace transform of a function (or signal). However, in MATLAB®, the Laplace transform $F(s)$ of a function $f(t)$ is easily computed by executing the command "*laplace*". Before using the command, the declaration of complex frequency s and of time t as symbolic variables is necessary. To define a symbolic variable, the command "*syms*" is used. Finally, note that the command "*laplace*" computes the unilateral Laplace transform of a function.

Functions $f(t) = e^{-t}$ and $F(s) = \frac{1}{s+1}$ are a Laplace transform pair. In other words, the (unilateral) Laplace transform of $f(t) = e^{-t}$ is $F(s) = \frac{1}{s+1}$. A Laplace transform pair is denoted by $f(t) \rightarrow F(s)$. In our case, the Laplace transform pair is $e^{-t} \rightarrow \frac{1}{s+1}$

Activity-1: Use MATLAB and compute the Laplace transform of the functions given below.

1. $x(t) = \delta(t)$

```
syms t
x=dirac(t);
X=laplace(x);
```

2. $x(t) = u(t)$

```
clear
clc
syms t
x=heaviside(t);
```

3. $x(t) = e^{-at}u(t)$

```
clear
clc
syms t a
x=exp(-a*t)*heaviside(t);
```

4. $x(t) = 5e^{-3t}u(t) + 2e^{3t}u(t)$

```
clear
clc
syms t
x=(5*exp(-3*t)*heaviside(t))+(2*exp(3*t)*heaviside(t));
```

5. $x(t) = \cos(\omega t) + \sin(\omega t)$

```
clear
clc
syms t w
x=cos(w*t)+sin(w*t);
```

Inverse Laplace Transform

The use of the one-sided Laplace transform is better suited for the purpose of this lab, as the signals considered are usually casual signals. In order to return from the s-domain back to the time domain, the inverse Laplace transform is applied. The inverse Laplace transform is denoted by the symbol $L^{-1}\{.\}$; that is, one can write

$$f(t) = L^{-1}\{F(s)\}$$

Or alternatively

$$F(s) \overset{L^{-1}}{\longrightarrow} f(t)$$

The mathematically expression that describes the inverse Laplace transform of a function $F(s)$ is:

$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)e^{st}dt$$

Command *ilaplace*

The command one uses now is "*ilaplace*". Before using the command, the declaration of complex frequency s and of time t as symbolic variables is necessary. To define a symbolic variable, the command "*syms*" is used.

Activity-2: Use MATLAB and compute the Inverse-Laplace transform of the functions given below. Compute partial fractions using "*residue*" where applicable.

1. $X(s) = \frac{5}{s+3} + \frac{3}{s-2}$

```
clear
clc
syms s
X=(5/(s+3))+(3/(s-2));
disp('X(s)= ');
```

2. $X(s) = \frac{s-1}{s^2+3s+2}$

```
clc
syms s
num=[1 -1];
denum=[1 3 2];
[r,p,k]=residue(num,denum)
X=(r(1)/(s-p(1)))+(r(2)/(s-p(2)));
disp('X(s)= ');
pretty(X);
disp('x(t)= ')
x=ilaplace(X);
disp(x);
disp('      *Direct conversion without converting to partial fraction*');
X=((s-1)/((s^2)+(3*s)+2));
disp('X(s)= ')
pretty(X)
disp('x(t)= ')
x=ilaplace(X); disp(x);
```

3. $X(s) = \frac{s^2-1}{s^2(s+2)}$

```
clear
clc
syms s
num=[1 0 1];
denum=[1 2 0 0];
[r,p,k]=residue(num,denum)
X=(r(1)/(s-p(1)))+(r(2)/s)+(r(3)/s^2);
disp('X(s)= ');
pretty(X);
disp('x(t)= ')
x=ilaplace(X);
```

```
pretty(x);
disp('    *Direct conversion without converting to partial fraction*');
X=((s^2+1)/(s^2*(s+2)));
disp('X(s)= ')
pretty(X)
disp('x(t)= ')
x=ilaplace(X);
pretty(x);
```

Lab Tasks (Psychomotor)

1. Compute the Laplace transform of the following signals using MATLAB.

   a. $\boldsymbol{x(t) = 3cos(2t) + sin(3\pi t)}$

   b. $\boldsymbol{x(t) = -1.25 + 3.5te^{-2t} + 1.25e^{-3t}}$

   c. $\boldsymbol{x(t) = sin(2t)cos(2t)u(t)}$

   d. $\boldsymbol{x(t) = e^{-at}cos(\omega t)u(t) + e^{-at}sin\,(\omega t)u(t)}$

2. Compute the Inverse-Laplace transform of the following signals using MATLAB. Compute partial fractions using "*residue*" command where applicable.

   1. $\boldsymbol{X(s) = \frac{1}{s(s+2)(s+3)}}$

   2. $\boldsymbol{X(s) = \frac{3s^2+2s+3}{s^2+3s+2}}$

   3. $\boldsymbol{X(s) = \frac{e^{-s}}{s+1} + \frac{e^{-2s}}{s+2}}$

3. Compute the convolution between the signals given below using Laplace transform in MATLAB.

$$x(t) = 5e^{-t}u(t), \qquad h(t) = te^{-t}u(t)$$

Lab Tasks (Cognitive)

Q. No.	Cognitive Assessment Question	Marks
1	Using standard Laplace-transform properties, determine the transform of 3cos(2t)+sin(3πt).	2
2	For X(s)=1/[s(s+2)(s+3)], identify the poles and state the type of time-domain terms expected in the inverse transform.	2
3	For X(s)=e^(−s)/(s+1), determine the time-domain effect represented by the factor e^(−s).	2
4	Using the convolution theorem, explain how the Laplace transforms of x(t)=5e^(−t)u(t) and h(t)=te^(−t)u(t) can be used to determine their convolution.	2
5	For a rational Laplace expression, explain why partial-fraction decomposition is useful before finding the inverse transform.	2

Lab Exercise and Summary

Summary should cover Introduction, Procedure, Data Analysis and Evaluation.

Student's Signature: _____ **Date: _____**

LABORATORY SKILLS ASSESSMENT (Cognitive)

Obtained Marks	/10

Instructor's Signature: _____ Date: _____

LABORATORY SKILLS ASSESSMENT (Psychomotor)

Psychomotor Activity 1 — Experimental Setup

S. No	Criteria	Performance	Description	Marks
1	**MATLAB Symbolic Setup (4)**	Level 4	Correctly prepares MATLAB with symbolic variables and the required Laplace-transform environment.	4
		Level 3	Prepares MATLAB with symbolic variables and the required Laplace-transform environment with minor errors/assistance.	3
		Level 2	Partially prepares MATLAB with symbolic variables and the required Laplace-transform environment with considerable assistance.	2
		Level 1	Largely incomplete MATLAB symbolic setup with substantial assistance.	1
2	**Transform File Setup (4)**	Level 4	Correctly organizes the script for forward Laplace, inverse Laplace, residue/ partial-fraction and convolution tasks.	4
		Level 3	Organizes the script for forward Laplace, inverse Laplace, residue/partial-fraction and convolution tasks with Minor errors/assistance	3
		Level 2	Organizes the script, partially completed and requires considerable assistance.	2
		Level 1	Shows limited ability to organizes the script and requires substantial assistance.	1
3	**Transform Parameters (2)**	Level 2	Correctly defines s, t and all coefficients, delays and signal parameters.	2
		Level 1	Defines s, t and all coefficients, delays and signal parameters with errors or assistance.	1
Total Obtained				/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Psychomotor Activity 2 — Implementation + Results

S. No	Criteria	Performance	Description	Marks
1	**Laplace Analysis Implementation & Safe Working Practice (4)**	Level 4	Correctly implements the assigned forward/inverse Laplace and convolution calculations independently and follows proper laboratory practices.	**4**
		Level 3	Implements the assigned work with minor errors or assistance and generally follows proper laboratory practices.	**3**
		Level 2	Partially implements the assigned work and requires considerable assistance or repeated guidance.	**2**
		Level 1	Implementation is largely incomplete, requires substantial assistance, or shows poor adherence to proper laboratory practices.	**1**
2	**Execution & Verification (2)**	Level 2	Correctly executes the symbolic commands and verifies the returned expressions.	**2**
		Level 1	Executes the symbolic commands and verifies the returned expressions with errors or assistance.	**1**
3	**Transform Results & Verification (4)**	Level 4	Obtains correct transform/time-domain expressions and verifies the assigned cases.	**4**
		Level 3	Obtains mostly correct transform /time-domain expressions and verifies the assigned cases with minor errors or omissions.	**3**
		Level 2	Obtains partially correct transform /time-domain expressions and verifies the assigned cases with noticeable errors or omissions.	**2**
		Level 1	Results/plots are largely incorrect or incomplete.	**1**
	Total Obtained			/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Instructor's Signature: _____ **Date:** _____

LABORATORY SKILLS ASSESSMENT (Affective)

Affective Activity — Lab Report + Viva Voce

S. No	Criteria	Performance	Description	Marks
1	**Lab Report Completeness & Organization (4)**	Level 4	Report is complete, well-organized, properly presented, and demonstrates responsible completion of the assigned Signals and Systems laboratory work.	**4**
		Level 3	Report is complete with minor omissions or organizational issues and demonstrates satisfactory responsibility for the assigned work.	**3**
		Level 2	Report is partially complete with several omissions and demonstrates limited responsibility for the assigned work.	**2**
		Level 1	Report has major omissions or poor organization and demonstrates minimal responsibility for the assigned work.	**1**
2	**Code, Results & Conclusion (2)**	Level 2	MATLAB code, results/plots, and conclusion are correctly presented and discussed, demonstrating ownership of the assigned work.	**2**
		Level 1	MATLAB code, results/plots, or conclusion are presented with minor errors or omissions.	**1**
3	**Viva Voce (4)**	Level 4	Clearly explains the assigned MATLAB code, signal/system concepts, analysis, results, and plots, demonstrating individual responsibility, participation, and effective communication.	**4**
		Level 3	Explains most of the assigned work and results correctly, with minor difficulty, and demonstrates good individual participation.	**3**
		Level 2	Explains basic aspects of the assigned work with some prompting and demonstrates basic participation.	**2**
		Level 1	Demonstrates limited understanding or involvement and requires substantial prompting to explain the assigned work.	**1**
	Total Obtained			/10

The student has not submitted the report and has not appeared in the Viva Voce.

Criteria	Description	Marks
Lab Report and Results Not Submitted	Student has not submitted the report and results file.	**0**
Absent in Viva Voce	Student is absent from the viva voce.	**0**

Instructor's Signature: _____ **Date:** _____