Experiment No. 3

Properties of Signals and their Transformations

Objectives

This experiment is designed to familiarize the students with the basic properties of continuous and discrete time signals. The main features of the experiment include:

1. Construction of periodic signals, casual signals, even and odd signals,
2. Operations on signals like time reversal or reflection time scaling and time shifting.

Equipment

1. PC
2. Installed MATLAB

Introduction to Properties of Signals

Periodic and Aperiodic Signals

A signal is said to be **periodic signal** if it has a definite pattern and repeats itself at a regular interval of time. Whereas, the signal which does not at the regular interval of time is known as an **aperiodic signal** or **non-periodic signal**.

Continuous Time Periodic Signal

A continuous time signal x(t) is said to be periodic if and only if

$$x(t + T) = x(t)\, for\, -\infty < t < \infty$$

Where, T is a positive constant that represents the time period of the periodic signal. The smallest value of the time period (T) which justifies the definition of the periodic signal is known as **fundamental time period** of the signal and is denoted by (T_o).

Also,

$$x(t + mT) = x(t)$$

Where, m is an integer. This means if the definition is satisfied for $T = T_o$, then it is also satisfied for $T = 2T_o$, $T = 3T_o$ … and so on with T_o as the fundamental time period. Therefore, the fundamental time period defines the duration of one complete cycle of the periodic signal x(t).

The reciprocal of the time period (T) of the periodic signal is called the frequency (f) of the signal, i.e.

$$f = \frac{1}{T}$$

Since the angular frequency is given by,

$$\omega = 2\pi f$$

$$\therefore \text{Time period, } T = \frac{2\pi}{\omega}$$

Some examples of continuous-time periodic signals are shown in Figure-1.

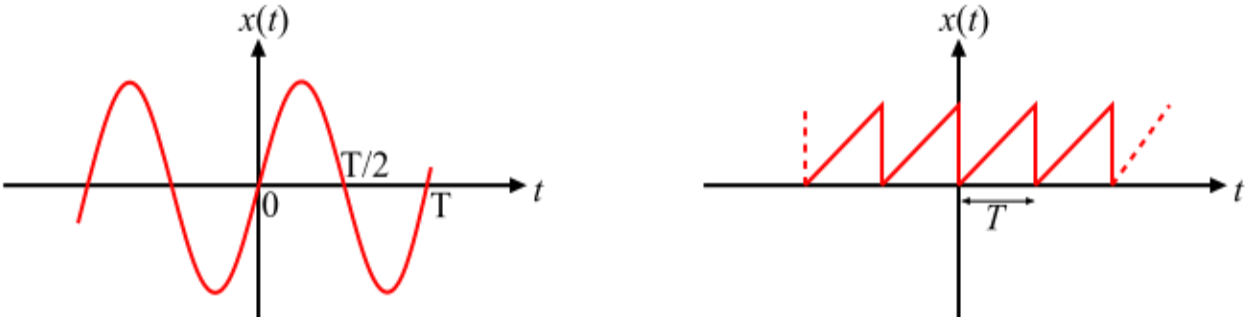


Figure 1: Continuous-time periodic signals

Discrete Time Periodic Signal

A discrete-time signal x(n) is said to be periodic if it satisfies the following condition

$$x(n) = x(n + N); \quad for\ all\ integers\ n$$

Here, N is the time period of the periodic signal and a positive integer. The smallest value of the time period (N) which satisfies the above condition is known as fundament time period of the signal. The fundamental time period (N) may be defined as the minimum number of samples taken by signal to repeat itself.

The angular frequency of the discrete time periodic sequences is given by,

$$\omega = \frac{2\pi}{N}$$

Therefore, the time period of the sequence is,

$$N = \frac{2\pi}{\omega}$$

Some examples of the discrete time periodic sequences are shown in Figure-2.

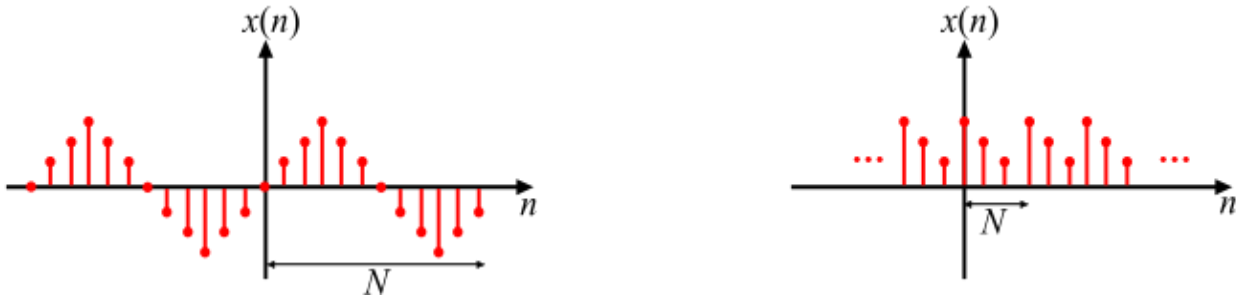


Figure 2: discrete time periodic sequences

Continuous Time Aperiodic Signal

If a continuous time signal does not have a definite pattern and does not repeat at regular intervals of time is known as continuous time **aperiodic signal** or **nonperiodic signal**.

In other words, a signal x(t) for which no value of time t satisfies the condition of periodicity, is known as **aperiodic** or **non-periodic signal**.

Some examples of continuous-time aperiodic signals is shown in Figure-3.

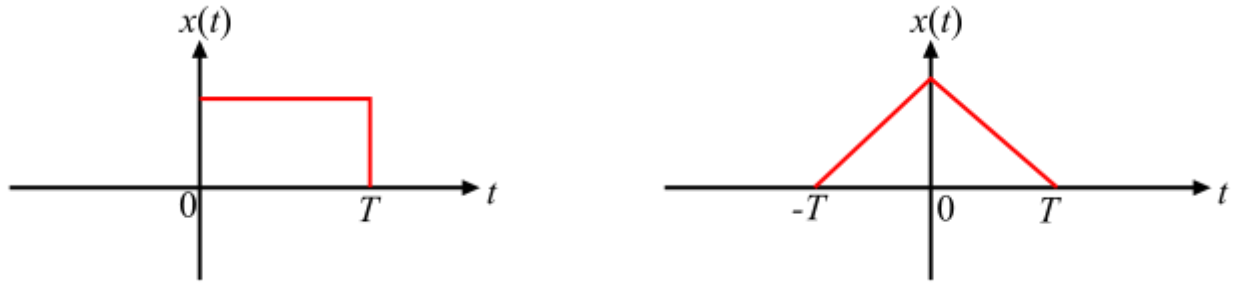


Figure 3: continuous-time aperiodic signals

Discrete Time Aperiodic Signal

If the condition of periodicity is not satisfied even for one value of n for a discrete time signal x(n), then the discrete time signal is aperiodic or nonperiodic. Examples of discrete time aperiodic signals are shown in Figure-4.

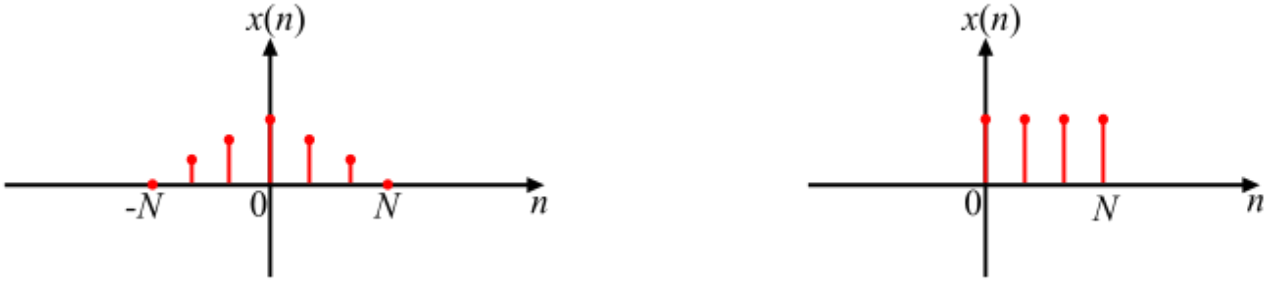


Figure 4: discrete time aperiodic signals

Activity-1: Examine whether the following signals are periodic or not? If periodic then determine the fundamental period of the signal.

$$\boldsymbol{x(t) = \sin(10\pi t)}$$

Solution

Since the signal x(t) is a sinusoidal signal, hence, it is a periodic signal.
Now, comparing x(t) with standard signal, i.e., sin ωt, we get,

$$\omega = 10\pi$$

$$\therefore Fundamental\ Period,\ T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi}$$

$$T = \frac{1}{5}\ sec$$

Causal, Non-Causal, and Anti-Causal Signals

Causal Signal

A continuous time signal $x(t)$ is called causal signal if the signal $x(t) = 0$ for $t < 0$. Therefore, a causal signal does not exist for negative time. The unit step signal u(t) is an example of causal signal as shown in Figure-5.

Similarly, a discrete time sequence x(n) is called the causal sequence if the sequence x(n) = 0 for n < 0.

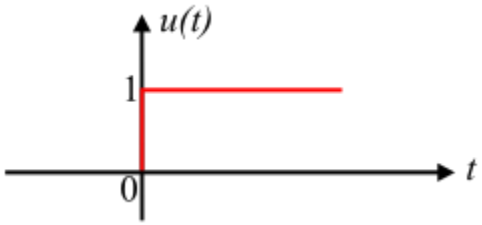


Figure 5: discrete time sequence

Anti-Causal Signal

A continuous-time signal x(t) is called the anti-causal signal if x(t) = 0 for t > 0. Hence, an anti-causal signal does not exist for positive time. The time reversed unit step signal u(-t) is an example of anti-causal signal (see Figure-6).

Similarly, a discrete time sequence x(n) is said to be anti-causal sequence if the sequence x(n) = 0 for $t > 0$.

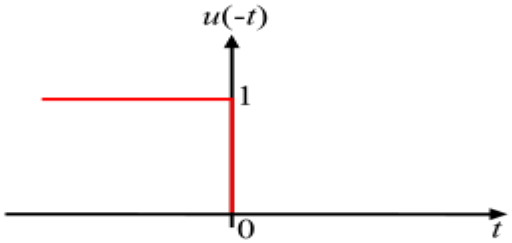


Figure 6: discrete time sequence x(n) to be anti-causal sequence

Non-Causal Signal

A signal which is not causal is called the non-causal signal. Hence, by the definition, a signal that exists for positive as well as negative time is neither causal nor anti-causal, it is non-causal signal. The sine and cosine signals are examples of non-causal signal (see Figure-7).

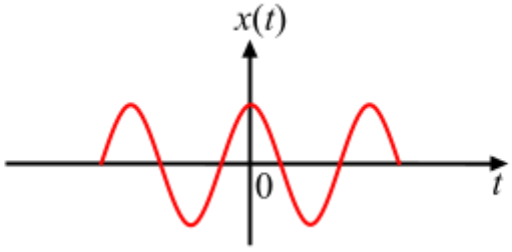


Figure 7: non-causal signal

Important – All the anti-causal signals are non-causal signals but the converse is not true.

Activity-2: Find which of the following signals are causal or anti-causal or non-causal.

$$x(t) = \sin(5t)\, u(t)$$

Solution

As the unit step signal u(t) does not exist for negative time. Thus, the signal x(t) is a causal signal. because x(t) = 0 for t < 0.

$$x(t) = 4u(-t)$$

Solution

The given signal x(t) exists only for negative time, i.e., t < 0. Therefore, it is an anti-causal signal. Also, it can be called non-causal signal.

$$x(n) = u(-n)$$

Solution

The given signal x(n) exists only for negative time, i.e., n < 0. Hence, it is anti-causal. It can also be called non-causal signal.

$$x(n) = u(n+3) - u(n-3)$$

Solution

The given signal x(n) exists from $n = -3$ to $n = 3$, i.e., the signal exists for both positive and negative time. Hence, it is a non-causal signal.

Even and Odd Signals

Even Signal

A signal which is symmetrical about the vertical axis or time origin is known as even signal or even function. Therefore, the even signals are also called the symmetrical signals. Cosine wave is an example of even signal.

Continuous-time Even Signal

A continuous-time signal x(t) is called the even signal or symmetrical signal if it satisfies the following condition,

$$x(t) = (-t); \text{ for } -\infty < t < \infty$$

Some examples of continuous-time even signals are shown in Figure-8.

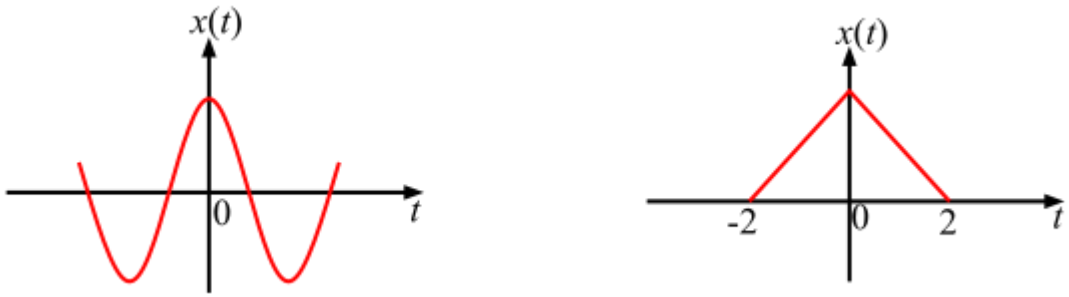


Figure 8: continuous-time even signals

Discrete-time Even Signal

A discrete-time signal x(n) is said to be even signal or symmetrical signal if it satisfies the condition,

$$x(n) = (-n); \text{ for } -\infty < n < \infty$$

Examples of discrete-time even signals are shown in Figure-9.

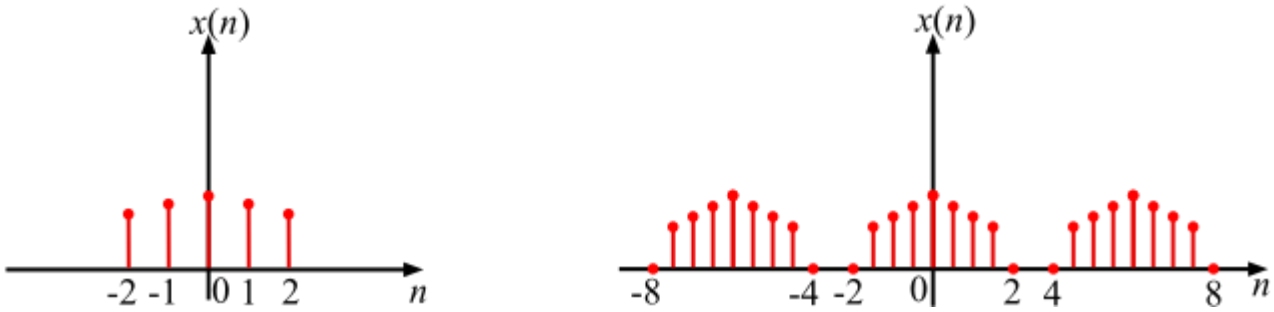


Figure 9: discrete-time even signals

Properties of Even Signals

The properties of the even signals are given as follows:

1. The even signals are symmetrical about the vertical axis.
2. The value of an even signal at time (t) is same as at time (-t).
3. The even signal is identical with its reflection about the origin.
4. Area under the even signal is two time of its one side area.

Odd Signal

A signal that is anti-symmetrical about the vertical axis is known as odd signal or **odd function**. Therefore, the odd signals are also called the **antisymmetric signals**. Sine wave is an example of odd signal.

Continuous-time Odd Signal

A continuous time signal x(t) is called an odd signal or antisymmetric signal if it satisfies the following condition,

$$x(-t) = -x(t); \text{ for } -\infty < t < \infty$$

Examples of continuous time odd signals or antisymmetric signals are shown in Figure-10.

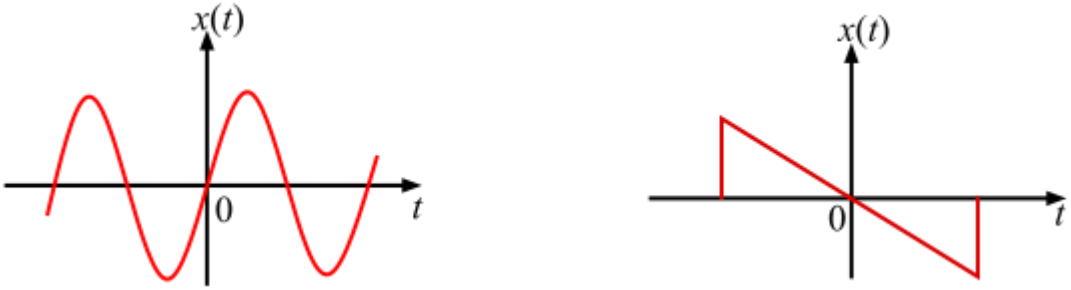


Figure 10: continuous time odd signals or antisymmetric signals

Discrete-time Odd Signal

A discrete time signal x(n) is said to be an odd signal or antisymmetric signal, if it satisfies the following condition,

$$x(-n) = -x(n); \text{ for } -\infty < n < \infty$$

Examples of discrete-time odd signals are shown in Figure-11.

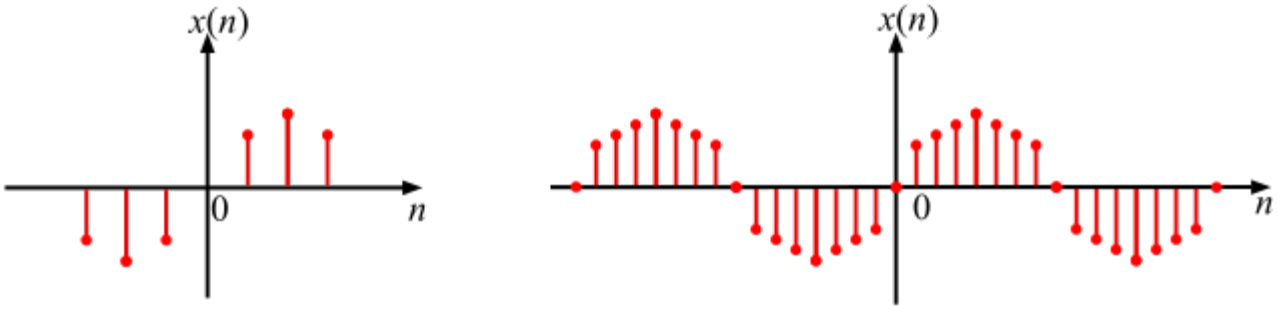


Figure 11: discrete-time odd signals

Properties of Odd Signals

Following are the properties of the odd signals:

1. The odd signal is antisymmetric about the origin.
2. The value of odd signal at time (t) is negative of its value at time (-t) for all t, i.e., $-\infty < t < \infty$.
3. The odd signal must necessarily be zero at time t = 0 to hold $x(0) = -x(0)$.
4. Area under the odd signal is always zero.

Note – A continuous-time signal is said to be **neither even nor odd** if it does not satisfy the condition of the even signal and that of the odd signal. Some examples of such signals (neither even nor odd) are shown in Figure-12.

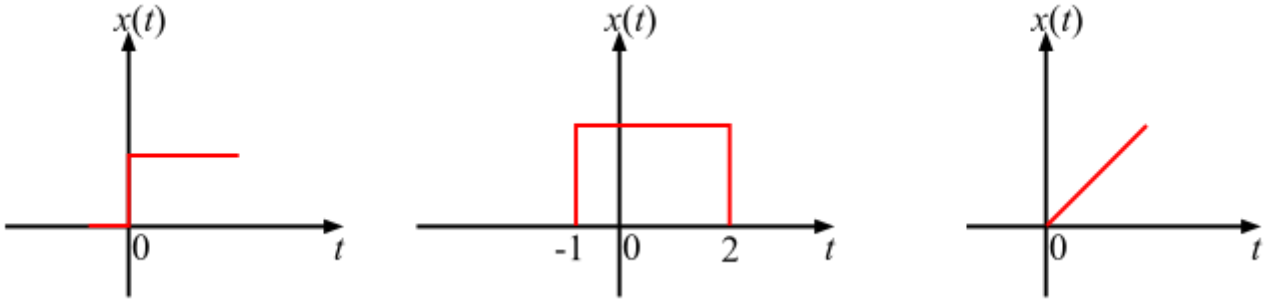


Figure 12: neither even nor odd signal

Activity-3: Find whether the signals are even or odd.

$$\boldsymbol{x(t) = sin(2t)}$$

Solution

The given signal is,

$$x(t) = \sin 2t$$

$$x(-t) = -\sin 2t$$

$$-x(t) = -\sin 2t$$

Hence, $(t) \neq (-t)$; but $x(-t) = -x(t)$, thus the given signal is an odd signal.

$$\boldsymbol{x(n) = cos(5t)}$$

Solution

Given signal is,

$$x(t) = \cos 5t$$

$$x(-t) = \cos 5t$$

$$-x(t) = -\cos 5t$$

Therefore, $(t) = (-t)$ and $x(-t) \neq -x(t)$, thus the given signal is an even signal.

Signal Transformations

Time Scaling of Signals

The process of multiplying a constant to the time axis of a signal is known as **time scaling of the signal**. The time scaling of signal may be time compression or time expansion depending

upon the value of the constant or scaling factor. The time scaling operation of signals is very useful when data is to be fed at some rate and is to be taken out at a different rate.

Time scaling of continuous-time signal

The time scaling of a continuous time signal x(t) can be accomplished by replacing 't' by 'αt' in the function. Mathematically, it is given by,

$$x(t) \rightarrow y(t) = x(\alpha t)$$

Where, α is a constant, called the scaling factor.

If α > 1, then the signal is compressed in time by a factor α and the time scaling of the signal is called the time compression. Whereas, if α < 1, then the signal is expanded in time by the factor α and the time scaling is said to be time expansion.

Example

Consider a continuous-time signal x(t) as shown in Figure-13. As it can be seen that the signal x(t) is increasing linearly from 0 to 5 in the time interval t = (-2) to t = 0 and remaining constant at 5 in the time interval t = 0 to t = 3 and then decreasing linearly from 5 to 0 in the time

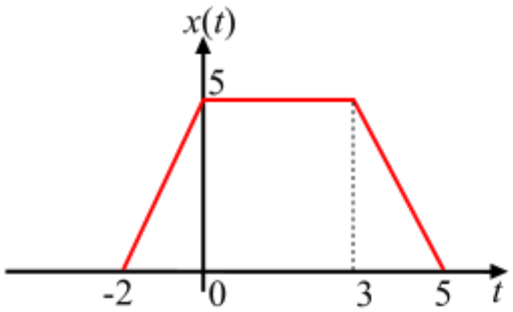


Figure 13

interval t = 3 to t = 5.

Case I – Consider the time scaling (compression) of the signal as,

$$(t) \rightarrow (t) = x(2t)$$

As the value of scaling factor α = 2, i.e., α > 1, thus there is the time compression of the signal and the compressed signal is shown in Figure-14.

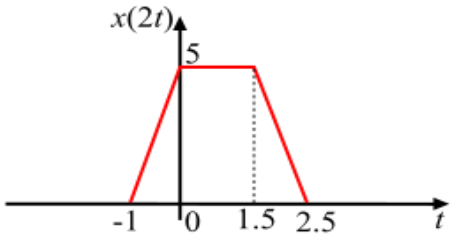


Figure 14

Here, it can be seen that the time compressed signal x(2t) increases linearly from 0 to 5 in the time interval $t = -(2/2)$ to $t = 0$ and remains constant at 5 in the time interval t = 0 to t = (3/2) and then decreases linearly from 5 to 0 in the time interval t = (3/2) to t = (5/2).

Case II – Consider the time scaling (expansion) of the signal as,

$$x(t) \rightarrow y(t) = x\left(\frac{t}{2}\right)$$

In this case, the value of α = (1/2), i.e., α < 1, hence there is the time expansion of the signal as shown in Figure-15.

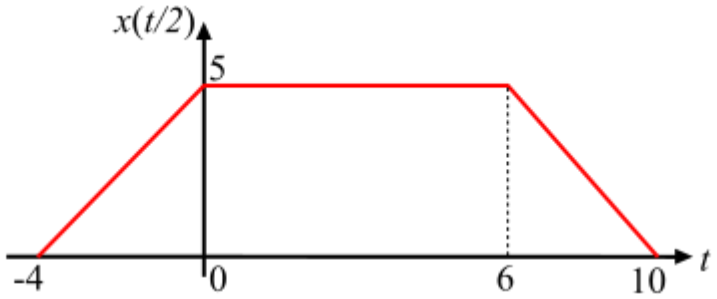


Figure 15

In Figure-3, the time expanded signal x(t/2) increases linearly from 0 to 5 in the time interval t = (-2×2) to t = 0, remains constant at 5 in the interval t = 0 to t = (3×2) and then decreases linearly from 5 to 0 in the interval t = (3×2) to t = (5×2).

Time Scaling of Discrete-Time Sequence

Consider a discrete time sequence x(n) as shown in Figure-16.

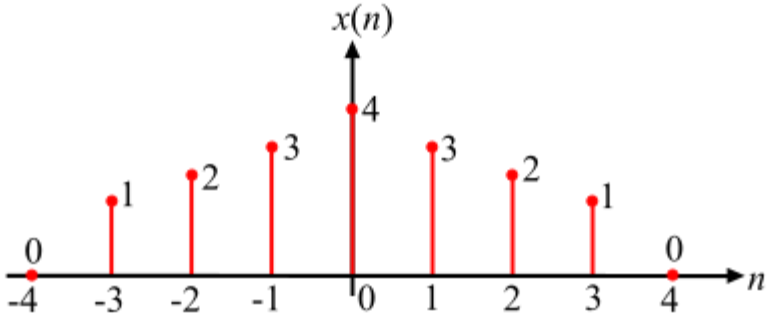


Figure 16

The time scaling for the discrete sequence can be defined as,

$$x(n) \rightarrow y(n) = x(kn)$$

Case I – If k = 2, then

$$(n) \rightarrow (n) = x(2n)$$

Here, k > 1, thus the signal is compressed in time. We can plot the time compressed signal y(n) by substituting different values of n as shown in Figure-17.

$$n = 0 \rightarrow (0) = (0) = 1$$

$$n = (-1) \rightarrow (-1) = (-2) = -2$$

$$n = (-2) \rightarrow (-2) = (-4) = 0$$

$$n = 1 \rightarrow (1) = (2) = 2$$

$$n = 2 \rightarrow (2) = (4) = 0$$

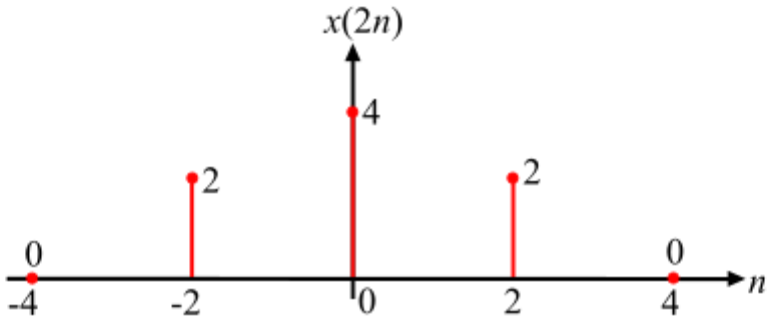


Figure 17

Therefore, to plot the signal x(2n), we have to skip the odd numbered samples in the signal x(n).

Case II – If k = (1/2), then

$$x(n) \rightarrow y(n) = x\left(\frac{n}{2}\right)$$

Since k < 1, hence the signal is expanded by a factor 2. We can plot the time expanded signal y(n) by substituting different values of n as follows −

$$n = 0 \rightarrow (0) = (0) = 1$$

$$n = 2 \rightarrow (2) = (1) = 3$$

$$n = 4 \rightarrow (4) = (2) = 2$$

$$n = (-2) \rightarrow (-2) = (-1) = 3$$

$$n = (-4) \rightarrow (-4) = (-2) = 2$$

In this case, all odd components in x(n/2) are zero because the signal x(n) does not have any value in between the sampling instants. Figure-18 shows the plot of the signal y(n)=x(n/2)

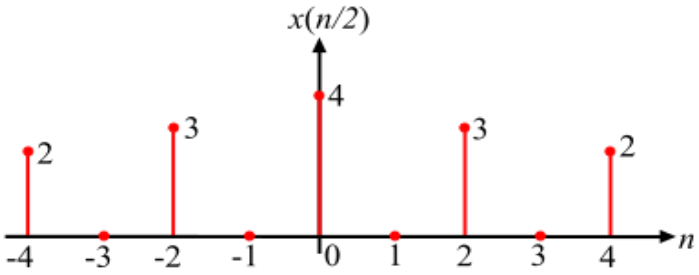


Figure 18

Amplitude Scaling of Signals

The process of rescaling the amplitude of a signal, i.e., the amplitude of the signal is either amplified or attenuated, is known as **amplitude scaling**. In the amplitude scaling operation on signals, the shape of the resulting signal remains the as that of the original signal but the amplitude is altered (i.e., increased or decreased).

Amplitude Scaling of a Continuous-Time Signal

The amplitude scaling of a continuous time signal x(t) is defined as,

$$y(t) = A\,x(t)$$

Where, A is a constant. If the value of A is greater than 1 (i.e., A > 1), the signal amplitude scaling is called the amplification of the signal while if A < 1, then the scaling is called the **attenuation of the signal**.

Figure-19 shows an arbitrary continuous-time signal x(t) and its amplitude scaled version y(t).

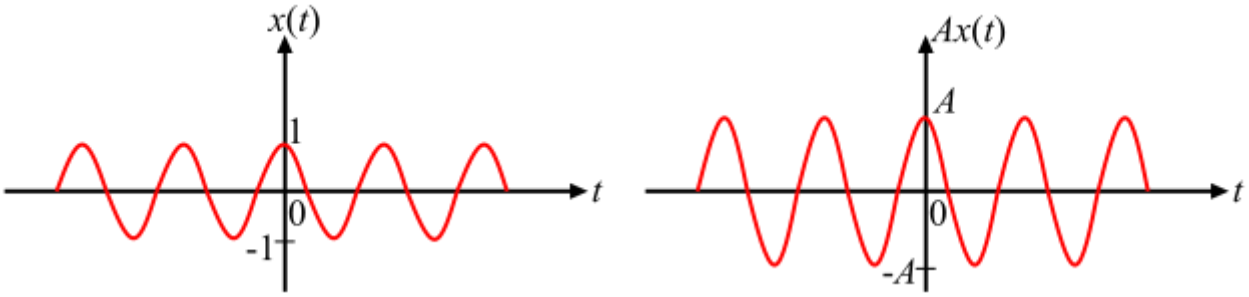


Figure 19

Activity-4: Sketch the following continuous-time signal.

$$\boldsymbol{x(t) = \Pi(t);\ y(t) = 2\,\Pi(t)}$$

Solution: The given signal is a unit rect function, i.e.,

$$(t) = \Pi(t)$$

The amplitude scaled signal [$(t) = 2\,\Pi(t)$] of the given signal is obtained by multiplying the amplitude of x(t) by 2 as shown in Figure-20. Here, it is to be noted that the value of scaling factor more than 1 (i.e., 2), hence the resulting signal will be an amplified signal.

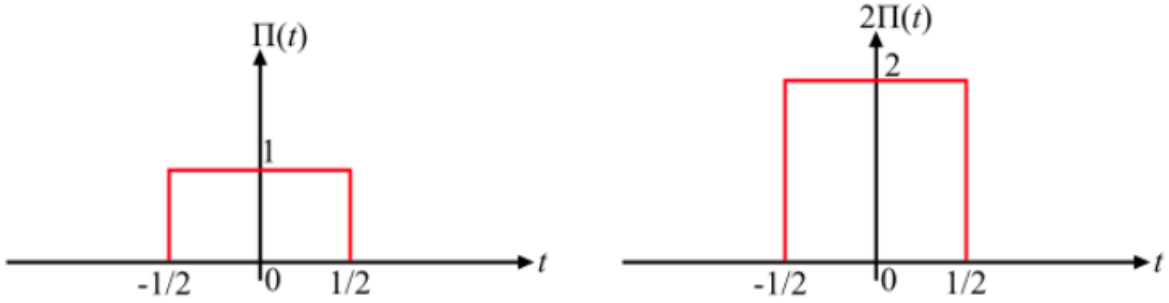


Figure 20

Amplitude Scaling of a Discrete-Time Signal

The amplitude scaling of a discrete time sequence x(n) is defined as,

$$y(n) = k\ x(n)$$

Where, k is a constant. If k > 1, the scaling is called amplification of the signal, while if k < 1, the scaling is called **attenuation of the signal**.

An arbitrary discrete time sequence x(n) and its scaled version y(n) are shown in Figure-21.



Figure 21

Lab Tasks (Psychomotor)

1. Verify that the given signal is periodic/aperiodic. Also plot three cycles of this signal.

$$\boldsymbol{y(t) = cos(t) + sin(3t)}$$

2. Verify that the given signal is periodic/aperiodic. Also plot three cycles of this signal.

$$\boldsymbol{y(n) = sin(\pi n + 2) + cos\left(\frac{2n}{3} + 1\right)}$$

3. Find whether the given signal is even or odd. Also write MATLAB code to plot the signal.

$$\boldsymbol{x(t) = e^{-5t}}$$

4. Write MATLAB script to plot the following continuous-time signal.

$$\boldsymbol{x(t) = u(t);\ y(t) = 2\,u(t)}$$

Lab Tasks (Cognitive)

Q. No.	Cognitive Assessment Question	Marks
1	For x(t)=cos(t)+sin(3t), determine whether the signal is periodic and find its fundamental period.	2
2	For y(n)=sin(πn+2)+cos(2n/3+1), determine whether a discrete-time fundamental period exists by applying the periodicity condition.	2
3	Given x(t)=e^(−5t), determine whether the signal is even, odd, or neither by applying x(−t) = x(t) and x(−t) = −x(t).	2
4	If y(t) = x(t−2), determine whether the operation represents a time delay or advance and state the amount.	2
5	For y(t) = 2x(t), determine whether the operation changes the amplitude, time scale, or both, and explain the effect of the factor 2.	2

Lab Exercise and Summary

Summary should cover Introduction, Procedure, Data Analysis and Evaluation.

Student's Signature: _____ **Date: _____**

LABORATORY SKILLS ASSESSMENT (Cognitive)

Obtained Marks	/10

Instructor's Signature: _____ Date: _____

LABORATORY SKILLS ASSESSMENT (Psychomotor)

Psychomotor Activity 1 — Experimental Setup

S. No	Criteria	Performance	Description	Marks
1	**Signal-Property Setup (4)**	Level 4	Correctly prepares MATLAB and the required signal representations for periodicity, symmetry and transformation operations.	4
		Level 3	Correctly prepares MATLAB and the required signal representations with minor errors or assistance.	3
		Level 2	Partially prepares the required environment and requires considerable assistance.	2
		Level 1	Shows limited ability to prepare MATLAB and requires substantial assistance.	1
2	**Transformation File Setup (4)**	Level 4	Correctly creates and organizes the script to generate original, reversed, shifted and scaled versions of the assigned signal.	4
		Level 3	Correctly creates and organizes the script to generate assigned signal with minor errors or assistance.	3
		Level 2	Partially completes the file setup and requires considerable assistance.	2
		Level 1	File setup is largely incomplete and requires substantial assistance.	1
3	**Time/Scaling Parameters (2)**	Level 2	Correctly defines time vector, shift amount and scaling factor.	2
		Level 1	Defines time vector, shift amount and scaling facto partially completed with considerable assistance.	1
Total Obtained				/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Psychomotor Activity 2 — Implementation + Results

S. No	Criteria	Performance	Description	Marks
1	**Signal Transformation Implementation & Safe Working Practice (4)**	Level 4	Correctly implements reversal, shifting, compression/expansion and required signal- property calculations independently and follows proper laboratory practices.	**4**
		Level 3	Implements the assigned work with minor errors or assistance and generally follows proper laboratory practices.	**3**
		Level 2	Partially implements the assigned work and requires considerable assistance or repeated guidance.	**2**
		Level 1	Implementation is largely incomplete, requires substantial assistance, or shows poor adherence to proper laboratory practices.	**1**
2	**Execution & Plotting (2)**	Level 2	Correctly executes the transformations and generates the required plots.	**2**
		Level 1	Executes the transformations and generates the required plots partially completed with considerable assistance.	**1**
3	**Property Verification (4)**	Level 4	Correctly verifies periodicity, causality or even/odd behavior and shows the effect of transformations.	**4**
		Level 3	Produces mostly correct property verification with minor errors or omissions.	**3**
		Level 2	Produces partially correct property verification with noticeable errors or omissions.	**2**
		Level 1	Property verifications are largely incorrect or incomplete.	**1**
	Total Obtained			/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Instructor's Signature: _____ **Date:** _____

LABORATORY SKILLS ASSESSMENT (Affective)

Affective Activity — Lab Report + Viva Voce

S. No	Criteria	Performance	Description	Marks
1	**Lab Report Completeness & Organization (4)**	Level 4	Report is complete, well-organized, properly presented, and demonstrates responsible completion of the assigned Signals and Systems laboratory work.	**4**
		Level 3	Report is complete with minor omissions or organizational issues and demonstrates satisfactory responsibility for the assigned work.	**3**
		Level 2	Report is partially complete with several omissions and demonstrates limited responsibility for the assigned work.	**2**
		Level 1	Report has major omissions or poor organization and demonstrates minimal responsibility for the assigned work.	**1**
2	**Code, Results & Conclusion (2)**	Level 2	MATLAB code, results/plots, and conclusion are correctly presented and discussed, demonstrating ownership of the assigned work.	**2**
		Level 1	MATLAB code, results/plots, or conclusion are presented with minor errors or omissions.	**1**
3	**Viva Voce (4)**	Level 4	Clearly explains the assigned MATLAB code, signal/system concepts, analysis, results, and plots, demonstrating individual responsibility, participation, and effective communication.	**4**
		Level 3	Explains most of the assigned work and results correctly, with minor difficulty, and demonstrates good individual participation.	**3**
		Level 2	Explains basic aspects of the assigned work with some prompting and demonstrates basic participation.	**2**
		Level 1	Demonstrates limited understanding or involvement and requires substantial prompting to explain the assigned work.	**1**
	Total Obtained			/10

The student has not submitted the report and has not appeared in the Viva Voce.

Criteria	Description	Marks
Lab Report and Results Not Submitted	Student has not submitted the report and results file.	**0**
Absent in Viva Voce	Student is absent from the viva voce.	**0**

Instructor's Signature: _____ **Date: _____**