Experiment No. 4

Linearity and Time Invariant Verification of LTI Systems

Objectives

This experiment is designed to familiarize the students with the basic concept of system properties. In this experiment following characteristics of the system will be addressed:

1. Linearity
2. Time Invariance

Equipment

1. PC
2. Installed MATLAB

Introduction to Properties of a System

In this section, we introduce the basic system properties. An illustrative example accompanies each property. Of course, an example does not prove a property. However, the illustrated examples are carefully selected in order to correspond with the introduced property. The following properties apply to both continuous-time and discrete-time systems.

Causal and Non-Causal System

Causal System

A system whose output or response at any time instant (t) depends only on the present and past values of the input but not on the future values of the input is called the **causal system**. For a causal system, the output or response does not begin before the input signal is applied. This is why a causal system is also called a **non-anticipated system**.

The causal systems are real time systems and they can be physically realized. For a causal system, the impulse response of the system is zero for negative time (i.e., t < 0) because the impulse signal δ(t) or δ(n) exists only at t = 0 or n = 0, i.e.,

$$h(t) = 0;\ t < 0 \ \ldots \text{ for continuous time system}$$

$$h(n) = 0;\ n < 0 \ \ldots \text{ for discrete time system}$$

A resistor is an example of a continuous time causal system. Some more examples of causal systems are given below.

$$y(t) = x(t-3) + 3x(t)$$

$$y(t) = tx(t)$$

$$y(n) = x(n-1) + x(n)$$

$$y(n) = nx(n)$$

Where, y(t) or y(n) and x(t) or x(n) are the output and input of the system respectively.

Non-Causal System

A system whose output or response at any time instant (t) depends upon future values of the input is called the **non-causal system**. The non-causal systems can generate an output before the input is given; hence they are also known as **anticipative systems**.

The non-causal systems do not exist in real time. Also, these systems are not physically realizable. The image processing systems are the examples of non-causal systems.

Some examples of the non-causal systems are given below.

$$y(t) = x(t + 3) + 2x(t)$$

$$y(t) = x^2(t) + tx(t + 3)$$

$$y(n) = x(n) + 3x(3n)$$

$$y(n) = x^2(n) + 2x(n + 1)$$

Activity-1: Find whether the given systems are causal or non-causal.

$$\boldsymbol{y(t) = x^2(t) + x(t - 3)}$$

Solution

Causality of the given system can be determined by considering different values of t as follows:

$$t = 0 \rightarrow y(0) = x^2(0) + x(-3)$$

$$t = (-2) \rightarrow y(-2) = x^2(-2) + x(-5)$$

$$t = 2 \rightarrow y(2) = x^2(2) + x(-1)$$

Hence, for all the values of t, the output depends only on the present and past values of the input. Thus, the given system is a causal system.

$$\boldsymbol{y(t) = x(3 - t) + x(t - 2)}$$

Solution

Causality of the given system can be determined by considering different values of t as follows:

$$t = 0 \rightarrow y(0) = x(3) + x(-2)$$

$$t = (-1) \rightarrow y(-1) = x(4) + x(-3)$$

$$t = 1 \rightarrow y(1) = x(2) + x(-1)$$

It is clear that for some values of t, the output of the system depends on the future values of the input. Hence, the given system is a non-causal system.

Static and Dynamic System

Static System

A system whose response or output is due to present input alone is known as **static system**. The static system is also called **the memory-less system**. For a static or memory-less system, the output of the system at any instant of time (t for continuous-time system or n for discrete-time system) depends only on the input applied at that instant of time (t or n), but not on the past or future values of the input. A purely resistive electrical circuit is an example of static system. Some examples of continuous-time and discrete-time static systems are given below.

$$c(t) = r(t)$$

$$c(t) = 2r^2(t)$$

$$c(n) = r(n)$$

$$c(n) = 5r^2(n)$$

Where, c(t) or c(n) and r(t) or r(n) are the output and input of the system respectively.

Dynamic System

A system whose response or output depends upon the past or future inputs in addition to the present input is called the dynamic system. The dynamic systems are also known as memory systems. Any continuous-time dynamic system can be described by a differential equation or any discrete-time dynamic system by a difference equation.

An electric circuit containing inductors and (or) capacitors is an example of dynamic system. Also, a summer or accumulator, a delay circuit, etc. are some examples of discrete-time dynamic systems.

Some more examples of dynamic systems are given below.

$$c(t) = r(t - 3)$$

$$c(t) = r(t) + r(t + 1)$$

$$c(n) = r(5n)$$

$$c(n) = r(n) + r(n + 5)$$

Activity-2: Find whether the given systems are static or dynamic.

$$\boldsymbol{y(t) = x(t - 4)}$$

Solution: From the equation of the system, it is clear that the output depends upon the past values of the input. Hence, the given system is a dynamic system.

$$\boldsymbol{y(n) = x(6n)}$$

Solution: From the equation of the system, it is clear that the output depends upon the past values of the input. Hence, the given system is a dynamic system.

Stable and Unstable System

Stable System

A system is called a BIBO **(bounded input bounded output) stable system** or simply **stable system**, if and only if every bounded input produces a bounded output. The output of a stable system does not change unreasonably.

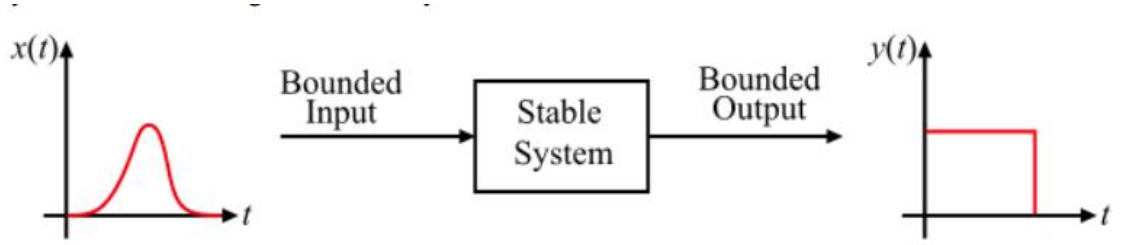


Figure 1: BIBO (bounded input bounded output) stable system)

The stability of a system indicates the usefulness of the system. The stability of a system can be determined from the impulse response of the system. The impulse response of the system is nothing but the output of the system for a unit impulse input.

If the impulse response of the system is absolutely integrable for a continuous time system or absolutely summable for a discrete time system, then the system is a stable system. Let an input signal x(t) is bounded signal, i.e.,

$$|x(t)| < K_x < \infty \text{ for } -\infty < t < \infty$$

Where, K_x is a positive real number. Then, if:

$$|y(t)| < K_y < \infty$$

That is, the output of the system y(t) is also bounded, then the system is called **BIBO stable system**.

Unstable System

If a system does not satisfy the BIBO stability condition, the system is called the unstable system. Therefore, for a bounded input, it is not necessary that the unstable system produces a bounded output. Thus, we can say that a system is unstable even if one bounded input generates an unbounded output.

Activity-3: Find whether the given systems are stale or unstable.

$$\boldsymbol{h(t) = \frac{1}{RC} e^{\left(\frac{-t}{RC}\right)} u(t)}$$

Solution: For stability of the system: $\int_{-\infty}^{\infty} |h(t)| < \infty$. For the given system,

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{\infty} \left| \frac{1}{RC} e^{\left(\frac{-t}{RC}\right)} u(t) \right| dt = \int_{0}^{\infty} \left| \frac{1}{RC} e^{\left(\frac{-t}{RC}\right)} \right| dt = 1 < \infty$$

Therefore, the given system is a stable system.

$$\boldsymbol{y(t) = (t + 7)u(t)}$$

Solution

$$y(t) = (t + 7);\ t \geq 0$$

Hence, for $t \to \infty$; $y(t) \to \infty$

Thus, the output of the system increases without any bound. Therefore, the given system is an unstable system.

Linear and Non-Linear System

Let $y(t)$ denotes the response of a system S to an input signal $x(t)$, that is , $y(t) = S\{x(t)\}$ System S is linear if for any input signals $x_1(t)$ and $x_2(t)$ and any scalars a_1 and a_2 the following relationship holds:

$$\boldsymbol{S\{a_1x_1(t) + a_2x_2(t)\} = a_1S\{x_1(t)\} + a_2S\{x_2(t)\}}$$

In other words, the response of a linear system to an input that is a linear combination of two signals is the linear combination of the responses of the system to each one of these signals. The linearity property is generalized for any number of input signals, and this is often referred to as the principle of superposition. The linearity property is a combination of two other properties the additivity property and the homogenous property. A system S satisfies the additivity property if for any input signals $x_1(t)$ and $x_2(t)$.

$$\boldsymbol{S\{x_1(t) + x_2(t)\} = S\{x_1(t)\} + S\{x_2(t)\}}$$

While the homogeneity property implies that for any scalar *a* and any input signal x(t):

$$\boldsymbol{S\{ax(t)\} = aS\{x(t)\}}$$

Activity-4: Check the linearity property for the given system. Use $-3 \leq t \leq 3$.

$$x_1(t) = u(t) - u(t-1), \qquad x_2(t) = u(t) - u(t-2), \qquad y(t) = 2x(t)$$

```
% Additive Property
clear
close all
t=-3:0.001:3;
x1=heaviside(t)-heaviside(t-1);
x2=heaviside(t)-heaviside(t-2);
S=2; y1=S*x1; y2=S*x2;
y3=y1+y2; y4=S*(x1+x2);
```

```
subplot(321),plot(t,x1);title('Input: x_1(t)=u(t)-u(t-1)'),xlabel('time(sec)');
subplot(322),plot(t,x2);title('Input: x_2(t)=u(t)-u(t-2)'),xlabel('time(sec)');
subplot(323),plot(t,y3);title(' Output: S[x_1(t)]+S[x_1(t)]'),xlabel('time(sec)');
subplot(324),plot(t,y4);title(' Output: S[x_1(t)+x_2(t)]'),xlabel('time(sec)');
% Homogenity Property
a=2;
x3=a*x1;
subplot(325),plot(t,S*x3);title('S[ax_1(t)]'),xlabel('time(sec)');
x4=S*x1;
subplot(326);plot(t,a*x4);title('aS[x_1(t)]'),xlabel('time(sec)');
```

Time Invariant-System

A system is time invariant, if a time shift in the input signal results in the same time shift in the output signal. In other words, if $y(t)$ is the response of a time invariant system to an input signal $x(t)$, then, the system response to the input signal $x(t-t_0)$ is $y(t-t_0)$. The mathematical expression is:

$$y(t-t_0) = S\{x(t-t_0)\}$$

Equivalently, a discrete-time system is time or (more appropriately) shift invariant if:

$$y(n-n_0) = S\{x(n-n_0)\}$$

From the above given expressions, we conclude that if a system is time invariant, the amplitude of the output signal is the same independent of the time instance the input is applied. The difference is a time shift in the output signal. A non-time-invariant system is called time-varying or time-variant system.

Activity-5: Check whether the given system is time-variant or time-invariant. Use time shift of 3 units along positive axis. Plot the outputs in the interval $-5 \leq t \leq 10$.

$$x(t) = u(t) - u(t-5), \qquad y(t) = te^{-t}x(t)$$

```
% Output Signal Shifting
clear
close all
t=-5:0.001:10;
x=heaviside(t)-heaviside(t-5);
```

```
y=t.*exp(-t).*x;

subplot(231),plot(t,x),title('Input: x(t)=u(t)-u(t-
5)'),xlabel('time(sec)');

subplot(232),plot(t,y);title('Output: y(t)=te^-
^tx(t)'),xlabel('time(sec)');

subplot(233),plot(t+3,y);title('Shifted Output: y(t-
3)'),xlabel('time(sec)');

% Input Signal Shifting

x=heaviside(t-3)-heaviside(t-8);

y=t.*exp(-t).*x;

subplot(234),plot(t,x);title('Shifted Input: x(t-3)=u(t-3)-u(t-
8)'),xlabel('time(sec)');

subplot(235),plot(t,y);title('Output with shifted input: y(t)=te^-^tx(t-
3)'),xlabel('time(sec)');
```

Lab Tasks (Psychomotor)

1. Find whether the given system is causal or non-causal.

$$\boldsymbol{y(n) = sin[x(n)]}$$

2. Find whether the given system is static or dynamic.

$$\boldsymbol{y(n) = x(n-1) + x(n)}$$

3. Find whether the given system is stable or not.

$$\boldsymbol{h(t) = e^{3t}u(t)}$$

4. Check the linearity of the following system. Plot the outputs in time interval $-3 \leq t \leq 3$.

$$\boldsymbol{x_1(t) = sin(2\pi f_1 t),} \qquad \boldsymbol{x_2(t) = sin(2\pi f_2 t),} \qquad \boldsymbol{y(t) = x(t)cos(3t)}$$

where, $f_1 = 1\ Hz$, $f_2 = 2\ Hz$

5. Consider a system given below and determine whether the system is time-invariant when the input signal is $x(t) = \cos(t)\,[u(t) - u(t-10)]$.

$$\boldsymbol{y(t) = 1 - 2x(t-1)}$$

Hint: $y(t) = 1 - 2cos(t-1)[u(t-1) - u(t-11)]$

Lab Tasks (Cognitive)

Q. No.	Cognitive Assessment Question	Marks
1	For y(n)=sin[x(n)], determine whether the system is causal by applying the dependence-on-input criterion.	2
2	For y(n)=x(n−1)+x(n), determine whether the system is static or dynamic and identify the term responsible for your conclusion.	2
3	For h(t)=e^(3t)u(t), determine whether the corresponding LTI system is stable by applying the absolute-integrability condition.	2
4	For y(t)=x(t)cos(3t), apply the superposition test to determine whether the system is linear.	2
5	For y(t)=1−2x(t−1), compare the output caused by a shifted input with the shifted original output and determine whether the system is time invariant.	2

Lab Exercise and Summary

Summary should cover Introduction, Procedure, Data Analysis and Evaluation.

Student's Signature: _____ **Date: _____**

LABORATORY SKILLS ASSESSMENT (Cognitive)

Obtained Marks	/10

Instructor's Signature: _____ Date: _____

LABORATORY SKILLS ASSESSMENT (Psychomotor)

Psychomotor Activity 1 — Experimental Setup

S. No	Criteria	Performance	Description	Marks
1	**System-Property Setup (4)**	Level 4	Correctly prepares MATLAB and the required input/system expressions for the assigned property-verification task.	4
		Level 3	Prepares MATLAB and the required input/system expressions for the assigned property-verification task with minor errors or assistance.	3
		Level 2	Partially prepares the required environment and requires considerable assistance.	2
		Level 1	Shows limited ability to prepare MATLAB and requires substantial assistance.	1
2	**Test-Case Setup (4)**	Level 4	Correctly creates organized script sections for causality, static/dynamic, stability, linearity and/or time-invariance tests.	4
		Level 3	Completes the Test-Case setup with minor errors or assistance.	3
		Level 2	Partially completes the Test-Case setup and requires considerable assistance.	2
		Level 1	Test-Case setup is largely incomplete and requires substantial assistance.	1
3	**Input & Shift Parameters (2)**	Level 2	Correctly defines input signals, scaling values and required time shifts.	2
		Level 1	Defines input signals, scaling values and required time shifts with minor errors or assistance.	1
Total Obtained				/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Psychomotor Activity 2 — Implementation + Results

S. No	Criteria	Performance	Description	Marks
1	System Property Implementation & Safe Working Practice (4)	Level 4	Correctly implements the required input-output relationships and property tests independently and follows proper laboratory practices.	4
		Level 3	Implements the assigned work with minor errors or assistance and generally follows proper laboratory practices.	3
		Level 2	Partially implements the assigned work and requires considerable assistance or repeated guidance.	2
		Level 1	Implementation is largely incomplete, requires substantial assistance, or shows poor adherence to proper laboratory practices.	1
2	Execution & Verification (2)	Level 2	Correctly executes the test cases and checks the resulting signals.	2
		Level 1	Executes the test cases and checks the resulting signals with minor errors or assistance.	1
3	Property Results & Verification (4)	Level 4	Correctly verifies the assigned system properties from the obtained outputs/plots.	4
		Level 3	Verifies the assigned system properties from the obtained outputs/plots with minor errors/ assistance.	3
		Level 2	Verifies the assigned system properties from the partially completed obtained outputs/plots with considerable assistance.	2
		Level 1	Verifies the assigned system properties from the largely incomplete obtained outputs/plots with substantial assistance.	1
	Total Obtained			/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Instructor's Signature: _____ **Date:** _____

LABORATORY SKILLS ASSESSMENT (Affective)

Affective Activity — Lab Report + Viva Voce

S. No	Criteria	Performance	Description	Marks
1	**Lab Report Completeness & Organization (4)**	Level 4	Report is complete, well-organized, properly presented, and demonstrates responsible completion of the assigned Signals and Systems laboratory work.	**4**
		Level 3	Report is complete with minor omissions or organizational issues and demonstrates satisfactory responsibility for the assigned work.	**3**
		Level 2	Report is partially complete with several omissions and demonstrates limited responsibility for the assigned work.	**2**
		Level 1	Report has major omissions or poor organization and demonstrates minimal responsibility for the assigned work.	**1**
2	**Code, Results & Conclusion (2)**	Level 2	MATLAB code, results/plots, and conclusion are correctly presented and discussed, demonstrating ownership of the assigned work.	**2**
		Level 1	MATLAB code, results/plots, or conclusion are presented with minor errors or omissions.	**1**
3	**Viva Voce (4)**	Level 4	Clearly explains the assigned MATLAB code, signal/system concepts, analysis, results, and plots, demonstrating individual responsibility, participation, and effective communication.	**4**
		Level 3	Explains most of the assigned work and results correctly, with minor difficulty, and demonstrates good individual participation.	**3**
		Level 2	Explains basic aspects of the assigned work with some prompting and demonstrates basic participation.	**2**
		Level 1	Demonstrates limited understanding or involvement and requires substantial prompting to explain the assigned work.	**1**
	Total Obtained			/10

The student has not submitted the report and has not appeared in the Viva Voce.

Criteria	Description	Marks
Lab Report and Results Not Submitted	Student has not submitted the report and results file.	**0**
Absent in Viva Voce	Student is absent from the viva voce.	**0**

Instructor's Signature: _____ **Date: _____**